MATHEMATICS
A Practical Odyssey
8e

JOHNSON • MOWRY

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A PRACTICAL ODYSSEY

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Mathematics: A Practical Odyssey,
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*Highlighted sections can be found at www.cengagebrain.com

OVERVIEW

The goal of *Mathematics: A Practical Odyssey* is to expose students to the utility, relevance, and beauty of mathematics in the context of every-day themes and across multiple disciplines, and to broaden the narrow view of mathematics that may come from an isolated study of algebra. The text incorporates many items of interest, including historical notes, articles, and discussions of contemporary issues along with many rich illustrations and fine art to demonstrate a wide range of topics. We believe that students who engage with the content in this text will have a broader outlook and understanding of the world around them, in the spirit of a true liberal arts education. They will benefit not only from the analytical tools and mathematical skills they practice and acquire, but from the references to important scientific research and discoveries, as well as works of literature, history, art, and politics.

The following list is meant to make the text's goals more concrete. Skim the list to learn more about typical outcomes for each chapter. Also use the list to identify areas that you might cover in your course.

- In Chapter 1, *Logic,* learn to analyze the validity of an argument.
- In Chapter 2, *Sets and Counting,* and Chapter 3, *Probability,* learn to understand the risks of inherited diseases and the outcomes associated with lotteries and bets.
- In Chapter 4, *Statistics,* learn to understand the accuracy and validity of a public opinion poll.
- In Chapter 5, *Finance,* learn the ins and outs of buying a house or car, and using student loans to finance your college education.
- In Chapter 6, *Voting and Apportionment,* learn that there is no perfect voting system or method of apportionment.
- In Chapter 7, *Number Systems and Number Theory,* learn about the origins of commonly used number systems and their applications and learn about the expression of the Fibonacci numbers in nature and the golden ratio art.
- In Chapter 8, *Geometry,* learn to understand its origins and applications.
- In Chapter 9, *Graph Theory,* learn to create networks and apply graph theory to schedules.
- In Chapter 10, *Exponential and Logarithmic Functions,* learn how populations grow, how radiocarbon dating works, how the Richter scale measures earthquakes, and how sound is measured in decibels.
- In Chapter 11, *Markov Chains,* learn how manufacturers can predict their products' success or failure in the marketplace.
- In Chapter 12, *Linear Programming,* learn how a small business can determine how to utilize limited resources to maximize its profit.
- In Chapter 13, *Calculus,* learn more about the subject and its uses. (*Note:* this chapter does not appear within the textbook, but via www.cengagebrain.com only.)

Course Level and Flexibility

Mathematics: A Practical Odyssey is written for the student who has a working knowledge of intermediate algebra, not the student who has a perfect mastery of it. The student is expected to use this knowledge to acquire critical thinking and quantitative reasoning skills. Even though some chapters are not algebra-based, it could be challenging for a student without background in intermediate algebra to succeed in a course using this book. While the specific skills students learn in intermediate algebra courses do not always directly apply, the chapters require a level of critical thinking and mathematical maturity more commonly found in students who have passed intermediate algebra.

This book contains a wide range of topics. As much as possible, it has been written so that its chapters are independent of each other. Instructors have wide latitude in selecting topics to cover and can therefore customize the course so that it is responsive to the needs of each instructor's individual classes and institution. In some cases, a chapter has a suggested core of key sections as well as a selection of optional sections that offer enrichment. These sections are not labeled as "core" and "optional" in the book; rather, this distinction is made in the Chapter Summaries that are found in the Instructor's Resource Manual (which can be accessed via the Instructor's Companion Site at www.cengagebrain.com).

While instructors may pick and choose chapters to suit the needs of the course, we do recommend that a few topics precede others. Please see the Instructor's Resource Manual posted on the Instructor Companion Site for more specifics on content connections and dependencies as outlined in the Chapter Summaries.

New to the Eighth Edition

In Chapter 1, *Logic:*

- a new section 1.6 on *Deductive Proof of Validity* appears. In this section, elementary valid argument forms and common rules of replacement are developed; formal proofs of validity are then constructed by applying the basic rules of inference.
- the four standard form categorical proposition types are now included: the universal affirmative, the universal negative, the particular affirmative, and the particular negative.
- all conditional statements of the form "p implies q" now use standard terminology in which p is referred to as the "antecedent" and q the "consequent."

In Chapter 3, *Probability:*

- hemophilia is now covered along with other inherited diseases.
- the treatment of Simpson's paradox has been expanded.

In Chapter 4, *Statistics:*

- the distinction between sample and population standard deviation is introduced and explored in new exercises.

In Chapter 5, *Finance:*

- there is a new focus on *student finance,* including student loans and maxed-out credit cards.

In Chapter 7, *Number Systems and Number Theory:*

- there is a new emphasis on how certain bases, including base four, base five, base eight, base ten, base twelve, and base twenty, correspond to humanity's historical use of different body parts in counting, and why base sixty is used to keep time and to measure angles.

Chapter 11, *Markov Chains,* has been rewritten so that the material is more accessible, and so that review topics more closely fit our *just-enough, just-in-time* review policy.

In many chapters, especially Chapter 3 on Probability, and Chapter 5 on Finance, discussions and examples were heavily streamlined and given more structure to support students.

Exercises that require that the student engage in hands-on activities are now included. These same exercises can be used for in-class activities.

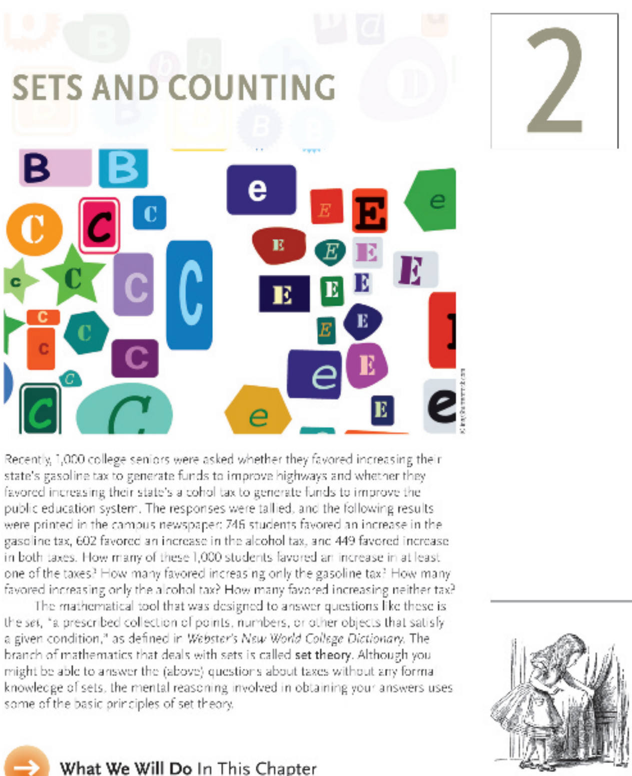
Throughout the textbook, real-world data within examples and exercises have been updated.

"Rough checks" now teach the student how to determine if an answer is reasonable; that is, if it's "in the ballpark."

Features

▶ Chapter Openers

The chapter openers provide background and motivation for the study of the topics within each chapter. "What We Will Do in This Chapter," gives a preview of concepts and topics that will be explored.



SETS AND COUNTING

2

Recently, 1,000 college seniors were asked whether they favored increasing their state's gasoline tax to generate funds to improve highways and whether they favored increasing their state's alcohol tax to generate funds to improve the public education system. The responses were tallied, and the following results were printed in the campus newspaper: 746 students favored an increase in the gasoline tax, 602 favored an increase in the alcohol tax, and 449 favored increase in both taxes. How many of these 1,000 students favored an increase in at least one of the taxes? How many favored increasing only the gasoline tax? How many favored increasing only the alcohol tax? How many favored increasing neither tax?

The mathematical tool that was designed to answer questions like these is the set, "a prescribed collection of points, numbers, or other objects that satisfy a given condition," as defined in *Webster's New World College Dictionary*. The branch of mathematics that deals with sets is called **set theory**. Although you might be able to answer the (above) questions about taxes without any formal knowledge of sets, the mental reasoning involved in obtaining your answers uses some of the basic principles of set theory.

What We Will Do In This Chapter

- We will use Venn diagrams to depict the relationships between sets. Is one set wholly contained within another, or, are there only a few, if any, common elements; if so, how many?

(continued)

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EXAMPLE 7 **GETTING FIVE HEARTS** Find the probability of being dealt five hearts.

SOLUTION The sample space is the same as in Example 5. The event consists of all possible five-card hands that include five hearts and no non-hearts. This involves two categories (hearts and non-hearts), so we will use the Fundamental Counting Principle and multiply the number of ways of getting five hearts and the number of ways of getting no non-hearts. There are

$${}_{13}C_5 = \frac{13!}{5! \cdot 8!} = 1{,}287$$

ways of getting five hearts, and there is

$${}_{39}C_0 = \frac{39!}{0! \cdot 39!} = 1$$

ways of getting no non-hearts. Thus, the probability of being dealt five hearts is

$$p(E) = \frac{{}_{13}C_5 \cdot {}_{39}C_0}{{}_{52}C_5} = \frac{1287 \cdot 1}{2{,}598{,}960} \approx 0.000495198 \approx 0.0005 = 1/2000$$

Rough Check In the event, there is a distinction between two categories (hearts and non-hearts); in the sample space, there is no such distinction. Thus, the numerator of

Calculation Notes

In Example 2, we do not use $t = 2 \text{ months} \cdot \frac{1 \text{ year}}{12 \text{ months}} = \frac{2}{12}$ years. If we did, we would get an inaccurate answer because some months are longer and some are shorter. Instead, we use the number of days, converted to years: $t = \frac{58}{365}$ years. This represents the time more accurately.

In Example 2, we naturally used 365 days per year, but some institutions traditionally count a year as 360 days and a month as 30 days (especially if that tradition works in their favor). This is a holdover from the days before calculators and computers—the numbers were simply easier to work with. Also, we used normal round-off rules to round \$187,131.301 . . . to \$187,131.30; some institutions round off some interest calculations in their favor. In this book, we will count a year as 365 days and use normal round-off rules (unless stated otherwise).

A written contract signed by the lender and the borrower is called a **loan agreement** or a **note**. The **maturity value** of the note (or just the **value** of the note) refers to the note's future value. Thus, the value of the note in Example 2 was \$187,131.30. This is what the note is worth to the lender in the future—that is, when the note matures.

◀ Examples

The worked-out examples contain detailed steps that effectively demonstrate solution methods. There is a special emphasis placed on checking that solutions are reasonable and presented in a form that is mathematically appropriate. Some of the checks are specially called out as "Rough Checks" to help students get in the habit of determining whether an answer makes sense. Other "Calculation Notes" provide tips to help students avoid common errors. Both types of checks can be found following the solutions of selected examples.

Featured in the News — To Many Americans, UFOS Are Real and Have Visited Earth in Some Form

Most Americans appear comfortable with and even excited about the thought of the discovery of extraterrestrial life. More than half (56 percent) of the American public think that UFOs are something real and not just in people's imagination. Nearly as many (48 percent) believe that UFOs have visited earth in some form. Males are significantly more likely to believe in the reality of UFOs, as are those under the age of 65. A significant drop is witnessed in the percentage of believers among the 65+ age group.

Two-thirds (67 percent) of adults think there are other forms of intelligent life in the universe. This belief tends to be more prevalent among males, adults ages 64 or younger, and residents of the Northeast as opposed to North Central and South.

In the view of many adults (55 percent), the government does not share enough information with the public in general. An even greater proportion (roughly seven in ten) thinks that the government does not tell us everything it knows about extraterrestrial life and UFOs. The younger the age, the stronger the belief that the government is withholding information about these topics.

This study was conducted by RoperASW. The sample consists of 1,021 male and female adults (in approximately equal number), all 18 years of age and over. The telephone interviews were conducted from August 23 through August 25, 2002, using a Random Digit Dialing (RDD) probability sample of all telephone households in the continental United States. The margin of error for the total sample is ±3 percent.

The Roper Poll UFOs & Extraterrestrial Life Americans' Beliefs and Personal Experiences (Prepared for the SCI FI Channel—September 2002)

◀ **Featured in the News**
Newspaper and magazine articles illustrate how the book's topics come up in the real world.

▶ **Topic X**
Topic X provides an opportunity to reinforce the practical emphasis of the text by illustrating current, powerful, real-word uses of the text's topics. The exercises that relate back to Topic X, included among the end-of-section exercises, take on increased significance and meaning as students see the text concepts put into direct context.

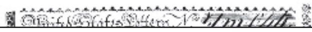
Topic X — The Business of Gambling: *Probabilities in the Real World*

It used to be that legal commercial gambling was not common. Nevada made casino gambling legal in 1931, and for more than thirty years, it was the only place in the United States that had legal commercial gambling. Then in 1964, New Hampshire instituted the first lottery in the United States since 1894. In 1978, New Jersey became the second state to legalize casino gambling. Now, state-sponsored lotteries are common, Native American tribes have casinos in more than half the states, and some states have casinos in selected cities or on riverboats.

It is very likely that you will be exposed to gambling if you have not been already. You should approach gambling with an educated perspective. If you are considering gambling, know what you are up against. The casinos all use probabilities and combinatorics in designing their games *to ensure that they make a consistent profit*. Learn this mathematics so that they do not take advantage of you.

Mountains. Benjamin Franklin used lotteries to finance cannons for the Revolutionary War, and John Hancock used lotteries to rebuild Faneuil Hall in Boston. Several universities, including Harvard, Dartmouth, Yale, and Columbia, were partly financed by lotteries. The U.S. Congress operated a lottery to help finance the Revolutionary War.

Today, public lotteries are a very big business. In 2012, Americans spent almost $66 billion on lottery tickets. Lotteries are quite lucrative for the forty-seven states that offer them; on the average, 30% of the money went back into government budgets. Fewer than half of the states dedicate the proceeds to education. Frequently, this money goes into the general fund. The states' cuts vary quite a bit. In Oregon, 54% of the money went to the state, and the remaining 46% went to prizes and administrative costs. In Rhode Island, the state took only 15%.

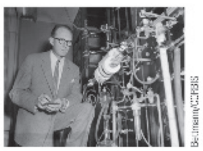


Most lottery sales come from a relatively small number of people. In Pennsylvania, for example, 29% of the players accounted for 79% of the spending on the lottery in 2008. However, many people who don't normally play go berserk when the jackpots accumulate, partially because of the amazingly large winnings but also because of a lack of understanding of how unlikely it is that they will actually win. The largest cumulative jackpot was $390 million, which was split by winners in Georgia and New Jersey in 2007. The largest single-winner jackpot was $315 million in West Virginia in 2002. The winner opted to take a lump sum payment of $114 million, instead of receiving twenty years of regular payments that would have added up to $315 million.

In this section, we will discuss probabilities and gambling. Specifically, we will explore lotteries, keno, and card games. See Examples 2–7, and Exercises 5–22 and 35.

In Section 3.5, we will discuss how

Historical Note — Willard Frank Libby, 1908–1980



Willard Frank Libby developed the radiocarbon dating technique in the mid-1940s. Carbon-14 was known to exist in nature, but little was known of its origins and properties. In 1939, Libby discovered that cosmic rays interacting with nitrogen at high altitudes produced a rapid formation of carbon-14. This high-altitude formation is the basis of the claim that the current ratio of carbon-14 to carbon-12 has been constant throughout history.

While working at the Enrico Fermi Institute of Nuclear Studies in Chicago, Libby was able to artificially produce carbon-14 and accurately determine its half-life. In addition, he devised a relatively simple device that measures the amount of carbon-14 in an organic sample. Before the creation of this device, measuring carbon-14 was a very expensive and difficult process. Libby's method made radiocarbon dating a practical possibility and revolutionized the fields of archeology and geology.

Libby received his doctorate degree in chemistry from the University of California at Berkeley in 1933 and taught there until 1945. During World War II, Libby also worked on the Manhattan Project, which developed the atomic bomb. During the years 1955–1959, he served on the U.S. Atomic Energy Commission, where he was instrumental in the formulation of many aspects of the commission.

After many years of dedicated research and numerous discoveries, Libby was awarded the Nobel Prize in chemistry in 1960 "for his method of using carbon-14 as a measurer of time in archeology, geology, geophysics, and other sciences."

◀ Found throughout the text, **Historical Notes** highlight the work of important people or events that relate to topics or concepts under discussion.

Exercises

The section-ending exercises are fresh, relevant, and span a wide variety of types. They are designed to solidify students' understanding of the material and make them proficient in the calculations involved.

In sections where *Topic X* appears, exercises relating back to the featured discussion are included.

3.4 EXERCISES

Use the guidelines on pages 181–182 to write each of your answers.

1. A group of thirty people is selected at random. What is the probability that at least two of them will have the same birthday?
2. A group of sixty people is selected at random. What is the probability that at least two of them will have the same birthday?
3. How many people would you have to have in a group so that there is a probability of at least 0.5 that at least two of them will have the same birthday?
4. How many people would you have to have in a group so that there is a probability of at least 0.9 that at least two of them will have the same birthday?
5. In 1990, California switched from a 6/49 lottery to a 6/53 lottery. Later, the state switched again, to a 6/51 lottery.

Concept questions test students understanding of ideas, often asking them to provide their own examples or explain main ideas in their own words.

b. If two people are selected at random, what is the probability that only one is a woman?
c. If two people are selected at random, what is the probability that both are men?
d. If you were an applicant and the two selected people were not of your gender, do you think that the above probabilities would indicate the presence or absence of gender discrimination in the hiring process? Why or why not?

32. Two hundred people apply for three jobs. Sixty of the applicants are women.
a. If three people are selected at random, what is the probability that all are women?

CONCEPT QUESTIONS

34. Explain why, in Example 6, $p(\text{four 2's}) = p(\text{four 3's}) = \cdots = p(\text{four kings}) = p(\text{four aces})$.
35. Do you think a state lottery is a good thing for the state's citizens? Why or why not? Be certain to include a discussion of both the advantages and disadvantages of a state lottery to its citizens.
36. Why are probabilities for most games of chance calculated with combinations rather than permutations?
37. Suppose a friend or relative of yours regularly spends (and loses) a good deal of money on lotteries. How would you explain to this person why he or

The short-answer **History** questions are meant to focus and reinforce the students' understanding of the historical material.

e. If you were an applicant and the three selected people were not of your gender, should the above probabilities have an impact on your situation? Why or why not?

33. In Example 2, $n(E) = 1$ because only one of the 7,059,052 possible lottery tickets is the first prize winner. Use combinations to show that $n(E) = 1$.

Answer the following questions using complete sentences and your own words.

HISTORY QUESTION

39. Are public lotteries relative newcomers to the American scene? Explain.

HANDS ON

40. What lottery games are available where you live? Choose one and find the probability of winning first, second, and third prizes.

f. Is there a pattern to the answers to parts (a)–(e)? If so, describe the pattern you see.
g. Use the pattern described in part (f) to predict the sum of the entries in the sixth row of Pascal's Triangle.
h. Find the sum of the entries in the sixth row of Pascal's Triangle. Was your prediction in part (g) correct?
i. Find the sum of the entries in the *n*th row of Pascal's Triangle.

46. a. Add adjacent entries of the sixth row of Pascal's Triangle to obtain the seventh row.
b. Find ${}_6C_r$ for $r = 0, 1, 2, 3, 4, 5,$ and 6.

62. For any given values of n and r, which is larger, ${}_nP_r$ or ${}_nC_r$? Why?

THE NEXT LEVEL

The following questions are modeled after those found on standardized entrance exams.

Exercises 63–67 refer to the following: A baseball league has six teams: A, B, C, D, E, and F. All games are played at 7:30 P.M. *on Fridays, and there are sufficient fields for each team to play a game every Friday night. Each team must play each other team exactly once, and the following conditions must be met:*

The Next Level These special exercises are designed to help prepare students for admission examinations such as the GRE (required for graduate school) or the GMAT (required for graduate study in business).

The **Hands On** exercises and **Projects** provide opportunites for further exploration or enrichment, often requiring students to try something first-hand or perform more open-ended research.

b. Find the line of best fit.
c. Predict the average mortgage rate if the median price of a home is $200,000.
d. Predict the median home price if the average mortgage rate is 5.25%.
e. Find the coefficient of correlation.
f. Are the predictions in parts (c) and (d) reliable? Why or why not?

15. What is a positive linear relation? Give an example.
16. What is a negative linear relation? Give an example.

PROJECTS

17. Measure the heights and weights of ten people. Let x = height and y = weight.
a. Plot the ordered pairs. Do the ordered pairs exhibit a linear trend?

Answers to the odd-numbered exercises are given in the back of the book, with two exceptions:

- Answers to historical questions and essay questions are not given.
- Answers are not given when the exercises instruct the student to check the answers themselves.

GRAPHING AND SCIENTIFIC CALCULATORS

Calculator boxes provide all of the necessary keystrokes for both scientific calculators and graphing calculators. Calculator subsections go beyond the keystrokes to offer more in-depth guidance.

Histograms on a Graphing Calculator

In Example 2 of this section, we created a frequency distribution and a his gram for the ages of the students in an acting class. Much of this work car done on a computer or a graphing calculator.

Technology and Statistical Graphs

Entering the Data

To enter the data from Example 2, do the following.

ENTERING THE DATA ON A TI-83/84:

- *Put the calculator into statistics mode* by pressing STAT.
- *Set the calculator up for entering the data* by selecting "Edit" from the "EDIT" menu, and the "list screen" appears, as shown in Figure 4.35. If data already appear in a list (as they do in list L_1 in Figure 4.35), use the arrow buttons to highlight the name of the list (i.e., "L_1" or "xStat") and press CLEAR ENTER.
- *Enter the students' ages* in list L_1, in any order, using the arrow buttons and the ENTER button. When this has been completed, your screen should look similar to that in Figure 4.35. Notice the "$L_1(43)=$" at the bottom of the screen; this indicates that 42 entries have been made, and the calculator is ready to receive the 43rd. This allows you to check whether you have left any entries out.

Note: If some data points frequently recur, you can enter the data points in list L_1 and their frequencies in list L_2 rather than reentering a data point each time it recurs.

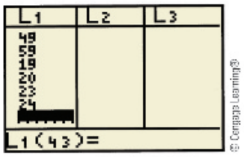


Figure 4.35 A TI-83/84's list screen.

The calculator boxes often offer keystrokes for several models of calculator, to provide optimal support.

First, we find the slope:

$$m = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2}$$
$$= \frac{5(1{,}058) - (57)(88)}{5(735) - (57)^2}$$
$$= 0.643192488\ldots$$

(5 × 1058 − 57 × 88) ÷ (5 × 735 − 57 x^2) =

For graphing calculators, see the instructions on page 330.

Once m has been calculated, we store it in the memory of our calculator. We will need it to calculate b, the y-intercept.

$$b = \bar{y} - m\bar{x}$$
$$= \left(\frac{88}{5}\right) - 0.643192488\left(\frac{57}{5}\right)$$
$$= 10.26760564\ldots$$

88 ÷ 5 − (RCL × 57 ÷ 5) =

Therefore, the line of best fit, $\hat{y} = mx + b$, is

Microsoft Excel©

A number of subsections that give instruction on the use of Excel are included. The following topics are addressed:

- Section 4.1: Histograms and Pie Charts on a Computerized Spreadsheet (page 251)
- Section 4.3: Measures of Central Tendency and Dispersion on Excel (page 285)
- Section 5.4: Amortized Loans (page 396)

These subsections allow instructors to incorporate the computer into their class if they so desire, but they are entirely optional, and the book is in no way computer dependent. The subsections do not assume any previous experience with Excel.

Amortization Schedules on Excel

Computing a year's amortization schedule is rather tedious, and neither a scientific calculator nor a graphing calculator offers relief. The best tool for the job is a computerized spreadsheet such as Excel. When you start Excel you see something that looks like a table waiting to be filled in. The rows are labeled with numbers and the columns are labeled with letters, as shown in Figure 5.24. The individual boxes are called *cells*. The cell in column A, row 1 is called cell A1.

	A	B	C	D	E	F	G
1	payment number	principal portion	interest portion	total payment	balance		
2	0				$175,000.00	rate	7.50%
3						years	15
4						payments/yr	12
5							

Figure 5.24 The spreadsheet after step 1.

We will discuss how to make an amortization schedule on Excel for a fifteen-year $175,000 loan at 7.5% interest with monthly payments.

1. *Set up the spreadsheet*. Start by entering the information shown in Figure 5.24.
   - Adjust the columns' widths. To make column A wider, place your cursor on top of the line dividing the "A" and "B" labels. Then use your cursor to move that dividing line.
   - Format cell E2 as currency by highlighting that cell and pressing the "$" button on the Excel ribbon at the top of the screen.
   - Be certain that you include the % symbol in cell G2.
   - Save the spreadsheet.
2. *Compute the monthly payment*. While you can do this with your calculator, as we discussed earlier in this section, you can also use the Excel "PMT" function to compute the monthly payment. This allows us to change the rate (in cell G2) or the loan amount (in cell E2) and Excel will automati-

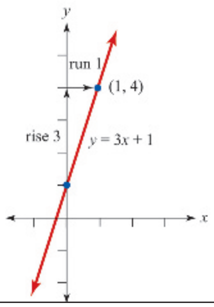
10.0A REVIEW OF EXPONENTIALS AND LOGARITHMS

OBJECTIVES

- Define and graph an exponential function; define the natural exponential function
- Use a calculator to find values of the exponential function 10^x and the natural exponential function e^x
- Define and understand the meaning of a logarithm
- Rewrite a logarithm as an exponential equation and vice versa
- Use a calculator to find values of the common and natural logarithmic functions $\log x$ and $\ln x$

This chapter will require a lot of calculator use. If your calculator skills are insufficient to the task, read Appendix A if you have a scientific calculator, or Appendix B if you have a graphing calculator.

Functions



An equation is said to be a **function** if to each value of x there corresponds one and only one value of y. For example, consider the equation $y = 3x + 1$; $x = 1$ corresponds to one and only one value of y (in particular, to $y = 4$). Other values of x also correspond to one and only one value of y, so the equation $y = 3x + 1$ is a function. When the value of y depends in this manner on the value of x, we say that y is a function of x; x is called the **independent variable,** and y is called the **dependent variable.**

Our equation $y = 3x + 1$ is called a *linear function,* because it is a function whose graph is a line. The graph of the equation $y = 3x + 1$ is shown in Figure 10.1. The slope of this line is 3; for every one-unit increase in x, the value of y increases by three units [slope = rise/run = (change in y)/(change in x)].

Algebra Review

Algebra reviews are *just-enough, just-in-time*. Algebra topics that may not have been covered in the students' algebra classes and topics to which students typically need multiple exposures are reviewed in sections placed immediately prior to the location where the topics are used, usually in a "section zero" at the beginning of the chapter. Those review sections cover only what is needed. More basic topics such as equation solving are not formally reviewed. Rather, the examples are selected so that they both explain the new material and review the appropriate algebra at the same time.

▶ Chapter Review and Review Exercises

At the end of every chapter is a Review including important Terms, Formulas, and summary of procedures or steps for solving problems found in the chapter. Following the Review are exercises similar to those found in each section of the chapter for students to use in preparation for a test or exam.

CHAPTER REVIEW

TERMS

bell-shaped curve
body table
categorical data
cells
coefficient of linear correlation
computerized spreadsheet
continuous variable
data point
density
descriptive statistics
deviation from the mean
discrete variable
error of the estimate
extreme value
frequency
frequency distribution
histogram
inferential statistics
level of confidence
line of best fit
linear regression
linear trend
margin of error (MOE)
mathematical model
mean
measures of central tendency
measures of dispersion
median
mode
normal distribution
outlier
pie chart
population
population proportion
relative frequency
relative frequency density
sample
sample proportion
spreadsheet
standard deviation
standard normal distribution
statistics
summation notation
tail
variance
z-distribution
z-number

REVIEW EXERCISES

1. Find (a) the mean, (b) the median, (c) the mode, and (d) the standard deviation of the following set of raw data:

   5 8 10 4 8 10 6 8 7 5

2. To study the composition of families in Winslow, Arizona, forty randomly selected married couples were surveyed to determine the number of children in each family. The following results were obtained:

   3 1 0 4 1 3 2 2 0 2 0 2 2 1
   4 3 1 1 3 4 2 1 3 0 1 0 2 5
   1 2 3 0 0 1 2 3 1 2 0 2

   a. Organize the given data by creating a frequency distribution.
   b. Find the mean number of children per family.
   c. Find the median number of children per family.
   d. Find the mode number of children per family.
   e. Find the standard deviation of the number of children per family.
   f. Construct a histogram using single-valued classes of data.

3. The frequency distribution in Figure 4.158 lists the number of hours per day that a randomly selected sample of teenagers spent watching television. Where possible, determine what percent of the teenagers spent the following number of hours watching television.

x = Hours per Day	Frequency
$0 \le x < 2$	23
$2 \le x < 4$	45
$4 \le x < 6$	53
$6 \le x < 8$	31
$8 \le x \le 10$	17

Figure 4.158 Time watching television.

   a. less than 4 hours
   b. not less than 6 hours
   c. at least 2 hours
   d. less than 2 hours
   e. at least 4 hours but less than 8 hours
   f. more than 3.5 hours

4. To study the efficiency of its new oil-changing system, a local service station monitored the amount of time it took to change the oil in customers' cars. The frequency distribution in Figure 4.159 summarizes the findings.

TO THE STUDENT, AS YOU EMBARK ON YOUR ODYSSEY . . .

This textbook is designed for students in liberal arts programs and other fields that do not require a core of mathematics. The term *liberal arts* is a translation of a Latin phrase that means “studies befitting a free person.” It was applied during the Middle Ages to seven branches of learning: arithmetic, geometry, logic, grammar, rhetoric, astronomy, and music. You might be surprised to learn that almost half of the original liberal arts are mathematics subjects.

In accordance with the tradition, handed down from the Middle Ages, that a broad-based education includes some mathematics, many institutions of higher education require their students to complete a college-level mathematics course. These schools award a bachelor’s degree to a person who not only has acquired a detailed knowledge of his or her field but also has a broad background in the liberal arts.

The goal of this textbook is to expose you to topics in mathematics that are usable and relevant to any educated person. We hope that you will encounter topics that will be useful at some time during your life. In addition, you are encouraged to recognize the relevance of mathematics to a well-rounded education and to appreciate the creative, human aspect of mathematics.

Your algebra background doesn’t have to be perfect, but algebra will come up. It’s also true that this book is written for a college-level math course. For the best result, you will have to work hard and put in a solid effort.

Your success in this course is important to us. To help you achieve that success, we have incorporated features in the textbook that promote learning and support various learning styles. Among these features are algebra review and instructions in using a calculator.

Our algebra reviews occur at the very beginning of the chapters, and they review only the algebra that comes up in that chapter. There are many topics in algebra in which students need to review and sharpen their skills. Calculator boxes give you all of the necessary keystrokes for scientific calculators and for graphing calculators. Calculator subsections help you learn how to use your calculator when a list of keystrokes is just not enough. We encourage you to examine these features and use them on your successful odyssey throughout this course.

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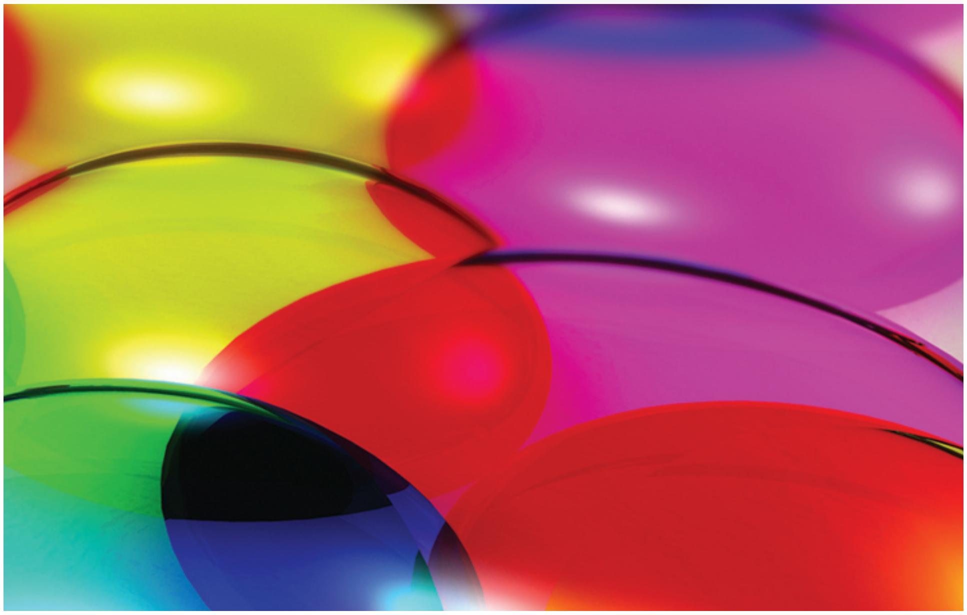
1

LOGIC



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When writer Lewis Carroll took Alice on her journeys down the rabbit hole to Wonderland and through the looking glass, she had many fantastic encounters with the tea-sipping Mad Hatter, a hookah-smoking Caterpillar, the Cheshire Cat, and Tweedledee and Tweedledum. On the surface, Carroll's writings seem to be delightful nonsense and mere children's entertainment. Many people are quite surprised to learn that *Alice's Adventures in Wonderland* is as much an exercise in logic as it is a fantasy and that Lewis Carroll was actually Charles Dodgson, an Oxford mathematician. In addition to "*Alice,*" Dodgson's many writings include the whimsical *The Game of Logic* and the brilliant *Symbolic Logic*.

Logic has fascinated scholars, philosophers, detectives, and star ship officers from the ancient, classic Greeks, to the ecentric, violin-playing Sherlock Holmes, to the deadpan, emotionless Mr. Spock. In today's world of misleading commercial claims, innuendo, and political rhetoric, it is imperative that we employ logic to distinguish valid from invalid arguments; consequently, armed with the fundamentals of logic, we can "live long and prosper."

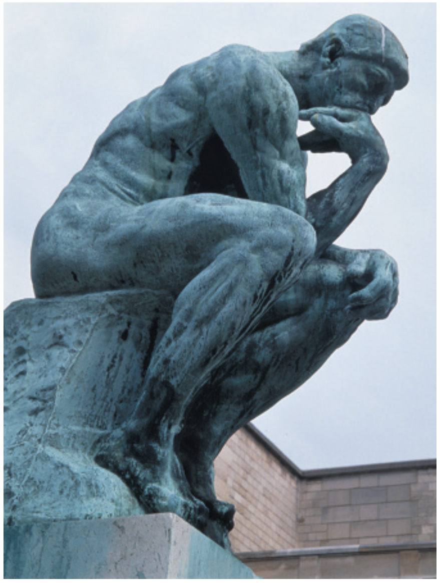
What We Will Do In This Chapter

- We will study the basic components of logic and its application.
- We will explore different types of reasoning (deductive and inductive) and how they are applied to puzzles, critical thinking, and problem solving.

(continued)

What We Will Do In This Chapter *(continued)*

- We will analyze and explore various types of statements and the conditions under which they are true. This process will be steamlined by translating verbal statements into symbolic form.
- We will explore equivalent forms of statements, including those of the conditional form "*if* . . . *then* . . .". In so doing, we will learn the difference between a "necessary condition" and a "sufficient condition."
- We will use various methods including Venn diagrams, truth tables, and formal deductive proofs to determine the validity of an argument.
- We will look at the lives and accomplishments of some of the influential people who have shaped the study logic.



Vanni Archive/Art Resource, NY

Logic is the science of correct reasoning. Auguste Rodin captured this ideal in his bronze sculpture *The Thinker.*

Paramount Television/The Kobal Collection/Picture Desk

In their quest for logical perfection, the Vulcans of *Star Trek* abandoned all emotion. Mr. Spock's frequent proclamation that "emotions are illogical" typified this attitude.

1.1 DEDUCTIVE VERSUS INDUCTIVE REASONING

OBJECTIVES

- Use Venn diagrams to determine the validity of deductive arguments
- Use inductive reasoning to predict patterns

Logic is the science of correct reasoning. *Webster's New World College Dictionary* defines **reasoning** as "the drawing of inferences or conclusions from known or assumed facts." Reasoning is an integral part of our daily lives; we take appropriate actions based on our perceptions and experiences. For instance, if you always encounter a traffic jam when taking a specific route while driving to school, you may decide to take an alternate route or leave home earlier on the day of an exam!

Problem Solving

Logic and reasoning are associated with the phrases *problem solving* and *critical thinking*. If we are faced with a problem, puzzle, or dilemma, we attempt to reason through it in hopes of arriving at a solution.

The first step in solving any problem is to define the problem in a thorough and accurate manner. Although this might sound like an obvious step, it is often overlooked. Always ask yourself, "What am I being asked to do?" Before you can solve a problem, you must understand the question. Once the problem has been defined, all known information that is relevant to it must be gathered, organized, and analyzed. This analysis should include a comparison of the present problem to previous ones. How is it similar? How is it different? Does a previous method of solution apply? If it seems appropriate, draw a picture of the problem; visual representations often provide insight into the interpretation of clues.

Before using any specific formula or method of solution, determine whether its use is valid for the situation at hand. A common error is to use a formula or method of solution when it does not apply. If a past formula or method of solution is appropriate, use it; if not, explore standard options and develop creative alternatives. Do not be afraid to try something different or out of the ordinary. "What if I try this . . . ?" may lead to a unique solution.


Avco Embassy/The Kobal Collection/Picture Desk

Using his extraordinary powers of logical deduction, Sherlock Holmes solves another mystery. "Finding the villain was elementary, my dear Watson."

Deductive Reasoning

Once a problem has been defined and analyzed, it might fall into a known category of problems, so a common method of solution may be applied. For instance, when one is asked to solve the equation $x^2 = 2x + 1$, realizing that it is a second-degree equation (that is, a quadratic equation) leads one to put it into the standard form ($x^2 - 2x - 1 = 0$) and apply the Quadratic Formula.

EXAMPLE 1

USING DEDUCTIVE REASONING TO SOLVE AN EQUATION Solve the equation $x^2 = 2x + 1$.

SOLUTION

The given equation is a second-degree equation in one variable. We know that all second-degree equations in one variable (in the form $ax^2 + bx + c = 0$) can be solved by applying the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Therefore, $x^2 = 2x + 1$ can be solved by applying the Quadratic Formula:

$$x^2 = 2x + 1$$

$$x^2 - 2x - 1 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - (4)(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2(1 \pm \sqrt{2})}{2}$$

$$x = 1 \pm \sqrt{2}$$

The solutions are $x = 1 + \sqrt{2}$ and $x = 1 - \sqrt{2}$.

In Example 1, we applied a general rule to a specific case; we reasoned that it was valid to apply the (general) Quadratic Formula to the (specific) equation $x^2 = 2x + 1$. This type of logic is known as **deductive reasoning**—that is, the application of a general statement to a specific instance.

Deductive reasoning and the formal structure of logic have been studied for thousands of years. One of the earliest logicians, and one of the most renowned, was Aristotle (384–322 B.C.). He was the student of the great philosopher Plato and the tutor of Alexander the Great, the conqueror of all the land from Greece to India. Aristotle's philosophy is pervasive; it influenced Roman Catholic theology through St. Thomas Aquinas and continues to influence modern philosophy. For centuries, Aristotelian logic was part of the education of lawyers and politicians and was used to distinguish valid arguments from invalid ones.

For Aristotle, logic was the necessary tool for any inquiry, and the syllogism was the sequence followed by all logical thought. A **syllogism** is an argument composed of two statements, or **premises** (the major and minor premises), followed by a **conclusion.** For any given set of premises, if the conclusion of an argument is guaranteed (that is, if it is inescapable in all instances), the argument is **valid.** If the conclusion is not guaranteed (that is, if there is at least one instance in which it does not follow), the argument is **invalid.**

Perhaps the best known of Aristotle's syllogisms is the following:

1. All men are mortal.	**major premise**
2. Socrates is a man.	**minor premise**
Therefore, Socrates is mortal.	**conclusion**

When the major premise is applied to the minor premise, the conclusion is inescapable; the argument is valid.

Notice that the deductive reasoning used in the analysis of Example 1 has exactly the same structure as Aristotle's syllogism concerning Socrates:

1. All second-degree equations in one variable can be solved by applying the Quadratic Formula.	**major premise**
2. $x^2 = 2x + 1$ is a second-degree equation in one variable.	**minor premise**
Therefore, $x^2 = 2x + 1$ can be solved by applying the Quadratic Formula.	**conclusion**

Each of these syllogisms is of the following general form:

1. If A, then B.	**All A are B. (major premise)**
2. x is A.	**We have A. (minor premise)**
Therefore, x is B.	**Therefore, we have B. (conclusion)**

Historically, this valid pattern of deductive reasoning is known as *modus ponens*.

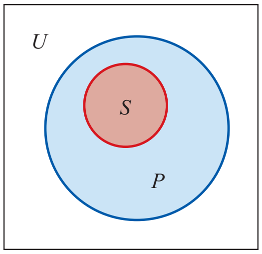


Figure 1.1 All S are P. (If S, then P.)

Deductive Reasoning and Venn Diagrams

The validity of a deductive argument can be shown by use of a Venn diagram. A **Venn diagram** is a diagram consisting of various overlapping figures contained within a rectangle (called U, the "universe"). To depict a statement of the form "All S are P" (or, equivalently, "If S, then P"), we draw two circles, one inside the other; the inner circle represents S (the "subject category") and the outer circle represents P (the "predicate category"). This relationship is shown in Figure 1.1.

Venn diagrams depicting "No S are P," "Some S are P," and "Some S are not P" are shown in Figures 1.2, 1.3, and 1.4, respectively.

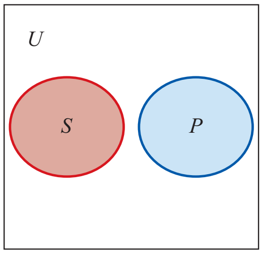


Figure 1.2 No S are P

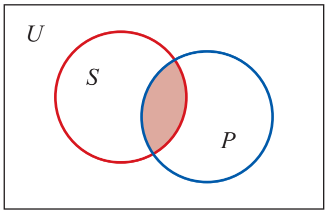


Figure 1.3 Some S are P. (At least one S is P.)

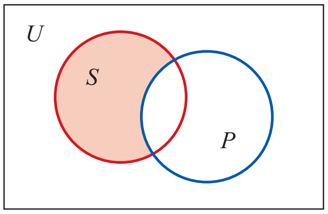


Figure 1.4 Some S are not P. (At least one S is not P.)

Categorical Proposition Type	Title	Venn Diagram
All S are P.	**Universal Affirmative**	Figure 1.1
No S are P.	**Universal Negative**	Figure 1.2
Some S are P.	**Particular Affirmative**	Figure 1.3
Some S are not P.	**Particular Negative**	Figure 1.4

Figure 1.5 Standard form categorical proposition types.

The four basic sentences (or propositions) "All S are P," "No S are P," "Some S are P," and "Some S are not P," are known as the standard form categorical proposition types. A **categorical proposition** is a statement affirming or denying that one category, S, (the subject category) is wholly or partially contained in some other category, P (the predicate category). Each of the standard form categorical propositions has a title, as indicated in Figure 1.5.

EXAMPLE 2

ANALYZING A DEDUCTIVE ARGUMENT Construct a Venn diagram to verify the validity of the following argument:

1. All men are mortal.
2. Socrates is a man.

Therefore, Socrates is mortal.

SOLUTION

Premise 1 is of the form "All A are B" and can be represented by a diagram like that shown in Figure 1.6.

Premise 2 refers to a specific man, namely, Socrates. If we let x = Socrates, the statement "Socrates is a man" can then be represented by placing x within the circle labeled "men," as shown in Figure 1.7. Because we placed x within the "men" circle, and all of the "men" circle is inside the "mortal" circle, the conclusion "Socrates is mortal" is inescapable; the argument is valid.

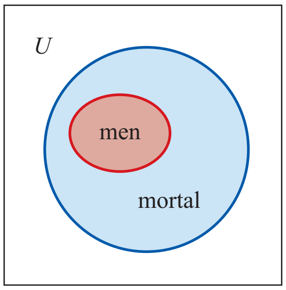


Figure 1.6 All men are mortal.

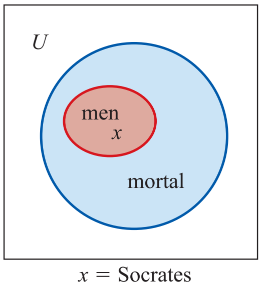
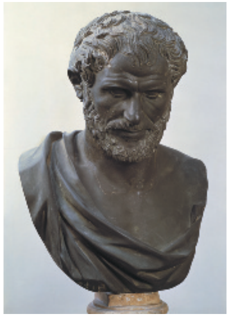


Figure 1.7 Socrates is mortal.

Historical Note Aristotle 384–322 B.C.



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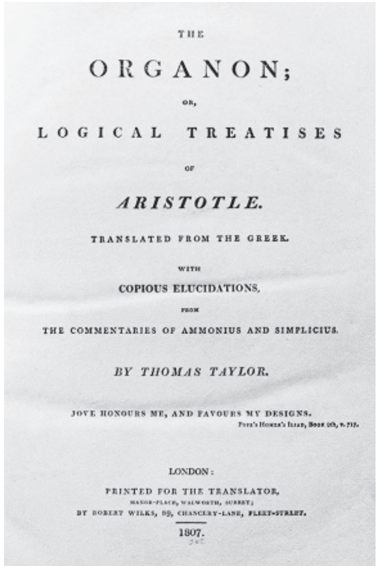
Aristotle was born in 384 B.C. in the small Macedonian town of Stagira, 200 miles north of Athens, on the shore of the Aegean Sea. Aristotle's father was the personal physician of King Amyntas II, ruler of Macedonia. When he was seventeen, Aristotle enrolled at the Academy in Athens and became a student of the famed Plato.

Aristotle was one of Plato's brightest students; he frequently questioned Plato's teachings and openly disagreed with him. Whereas Plato emphasized the study of abstract ideas and mathematical truth, Aristotle was more interested in observing the "real world" around him. Plato often referred to Aristotle as "the brain" or "the mind of the school." Plato commented, "Where others need the spur, Aristotle needs the rein."

Aristotle stayed at the Academy for twenty years, until the death of Plato. Then the king of Macedonia invited Aristotle to supervise the education of his son Alexander, the future Alexander the Great. Aristotle accepted the invitation and taught Alexander until he succeeded his father as ruler. At that time, Aristotle founded a school known as the Lyceum, or Peripatetic School. The school had a large library with many maps, as well as botanical gardens containing an extensive collection of plants and animals. Aristotle and his students would walk about the grounds of the Lyceum while discussing various subjects (*peripatetic* is from the Greek word meaning "to walk").

Many consider Aristotle to be a founding father of the study of biology and of science in general; he observed and classified the behavior and anatomy of hundreds of living creatures. Alexander the Great, during his many military campaigns, had his troops gather specimens from distant places for Aristotle to study.

Aristotle was a prolific writer; some historians credit him with the writing of over 1,000 books. Most of his works have been lost or destroyed, but scholars have re-created some of his more influential works, including *Organon*.



THE

ORGANON;

OR,

LOGICAL TREATISES

OF

ARISTOTLE.

TRANSLATED FROM THE GREEK.

WITH

COPIOUS ELUCIDATIONS,

FROM

THE COMMENTARIES OF AMMONIUS AND SIMPLICIUS.

BY THOMAS TAYLOR.

JOVE HONOURS ME, AND FAVOURS MY DESIGNS.

LONDON:

PRINTED FOR THE TRANSLATOR,

BY ROBERT WILKS, 89, CHANCERY-LANE, FLEET-STREET.

1807.

Collections of the New York Public Library

Aristotle's collective works on syllogisms and deductive logic are known as Organon, *meaning "instrument," for logic is the instrument used in the acquisition of knowledge.*

EXAMPLE 3

ANALYZING A DEDUCTIVE ARGUMENT Construct a Venn diagram to determine the validity of the following argument:

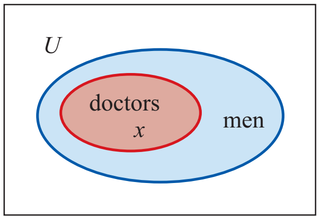
1. All doctors are men.
2. My mother is a doctor.

Therefore, my mother is a man.

SOLUTION

Premise 1 is of the form "All A are B"; the argument is depicted in Figure 1.8.

No matter where x is placed within the "doctors" circle, the conclusion "My mother is a man" is inescapable; the argument is valid.



x = My mother

Figure 1.8 My mother is a man.

Saying that an argument is valid does not mean that the conclusion is true. The argument given in Example 3 is *valid*, but the conclusion is *false*. One's mother cannot be a man! Validity and truth do not mean the same thing. An argument is valid if the conclusion is inescapable, *given the premises*. Nothing is said about the truth of the premises. Thus, when examining the validity of an argument, we are not determining whether the conclusion is true or false. Saying that an argument is valid merely means that, *given the premises,* the reasoning used to obtain the conclusion is logical. However, if the premises of a valid argument are true, then the conclusion will also be true.

EXAMPLE 4

ANALYZING A DEDUCTIVE ARGUMENT Construct a Venn diagram to determine the validity of the following argument:

1. All professional wrestlers are actors.
2. The Rock is an actor.

Therefore, The Rock is a professional wrestler.

SOLUTION

Premise 1 is of the form "All A are B"; the "circle of professional wrestlers" is contained within the "circle of actors." If we let x represent The Rock, premise 2 simply requires that we place x somewhere within the actor circle; x could be placed in either of the two locations shown in Figures 1.9 and 1.10.

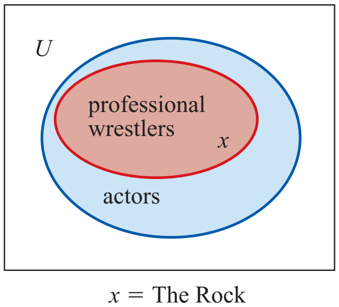


Figure 1.9

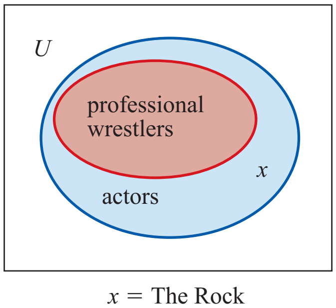
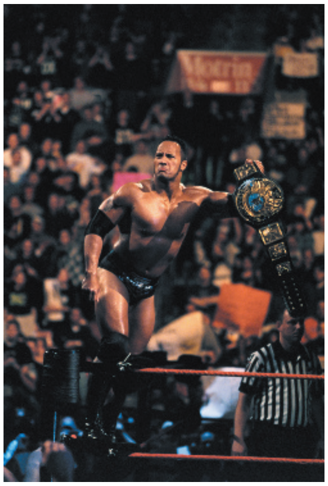


Figure 1.10

If x is placed as in Figure 1.9, the argument would appear to be valid; the figure supports the conclusion "The Rock is a professional wrestler." However, the placement of x in Figure 1.10 does not support the conclusion; given the premises, we cannot *logically* deduce that "The Rock is a professional wrestler." Since the conclusion is *not* inescapable, the argument is invalid.



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Even though The Rock *is* a professional wrestler, the argument used to obtain the conclusion is invalid.

Saying that an argument is invalid does not mean that the conclusion is false. Example 4 demonstrates that an invalid argument can have a true conclusion; even though The Rock (Dwayne Johnson) is a professional wrestler, the argument used to obtain the conclusion is invalid. In logic, validity and truth do not have the same meaning. *Validity* refers to the process of reasoning used to obtain a conclusion; *truth* refers to conformity with fact or experience.

Venn Diagrams and Invalid Arguments

To show that an argument is invalid, you must construct a Venn diagram in which the premises are met yet the conclusion does not necessarily follow.

EXAMPLE 5

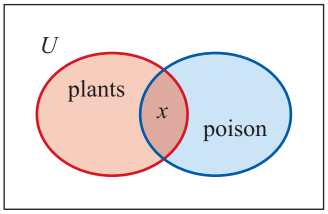
ANALYZING A DEDUCTIVE ARGUMENT Construct a Venn diagram to determine the validity of the following argument:

1. Some plants are poisonous.
2. Broccoli is a plant.

Therefore, broccoli is poisonous.

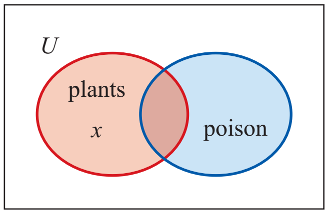
SOLUTION

Premise 1 is of the form "Some A are B"; it can be represented by two overlapping circles (as in Figure 1.3). If we let x represent broccoli, premise 2 requires that we place x somewhere within the plant circle. If x is placed as in Figure 1.11, the argument would appear to be valid. However, if x is placed as in Figure 1.12, the conclusion does not follow. Because we can construct a Venn diagram in which the premises are met yet the conclusion does not follow (Figure 1.12), the argument is invalid.



x = broccoli

Figure 1.11



x = broccoli

Figure 1.12

When analyzing an argument via a Venn diagram, you might have to draw three or more circles, as in the next example.

EXAMPLE 6

ANALYZING A DEDUCTIVE ARGUMENT Construct a Venn diagram to determine the validity of the following argument:

1. No snake is warm-blooded.
2. All mammals are warm-blooded.

Therefore, snakes are not mammals.

SOLUTION

Premise 1 is of the form "No *A* are *B*"; it is depicted in Figure 1.13. Premise 2 is of the form "All *A* are *B*"; the "mammal circle" must be drawn within the "warm-blooded circle." Both premises are depicted in Figure 1.14.

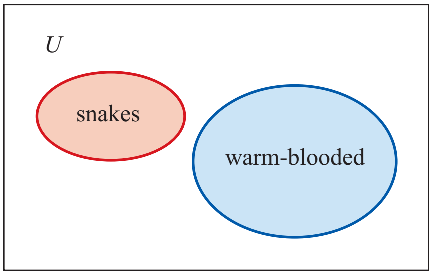
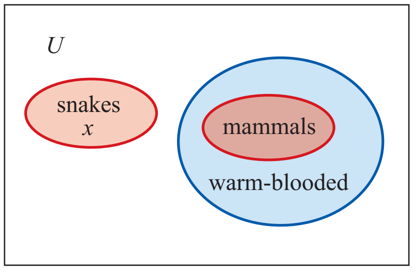


Figure 1.13 No snake is warm-blooded.



x = snake

Figure 1.14 All mammals are warm-blooded.

Because we placed x (= snake) within the "snake" circle, and the "snake" circle is outside the "warm-blooded" circle, x cannot be within the "mammal" circle (which is inside the "warm-blooded" circle). Given the premises, the conclusion "Snakes are not mammals" is inescapable; the argument is valid.

You might have encountered Venn diagrams when you studied sets in your algebra class. The academic fields of set theory and logic are historically intertwined; set theory was developed in the late nineteenth century as an aid in the study of logical arguments. Today, set theory and Venn diagrams are applied to areas other than the study of logical arguments; we will utilize Venn diagrams in our general study of set theory in Chapter 2.

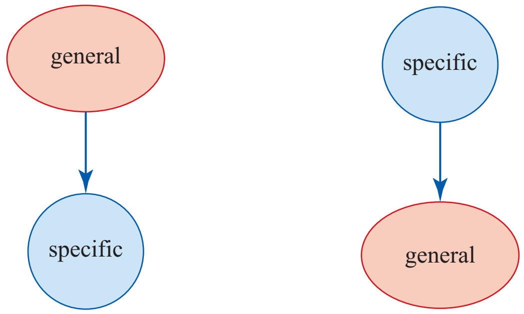
Inductive Reasoning

The conclusion of a valid deductive argument (one that goes from general to specific) is guaranteed: Given true premises, a true conclusion must follow.

However, there are arguments in which the conclusion is not guaranteed even though the premises are true. Consider the following:

1. Joe sneezed after petting Frako's cat.
2. Joe sneezed after petting Paulette's cat.

Therefore, Joe is allergic to cats.



Deductive Reasoning
(Conclusion is guaranteed.)

Inductive Reasoning
(Conclusion may be probable but is not guaranteed.)

Figure 1.15

Is the conclusion guaranteed? If the premises are true, they certainly *support* the conclusion, but we cannot say with 100% certainty that Joe is allergic to cats. The conclusion is *not* guaranteed. Maybe Joe is allergic to the flea powder that the cat owners used; maybe he is allergic to the dust that is trapped in the cats' fur; or maybe he has a cold!

Reasoning of this type is called inductive reasoning. **Inductive reasoning** involves going from a series of specific cases to a general statement (see Figure 1.15). Although it may seem to follow and may in fact be true, *the conclusion in an inductive argument is never guaranteed.*

EXAMPLE 7

INDUCTIVE REASONING AND PATTERN RECOGNITION What is the next number in the sequence 1, 8, 15, 22, 29, . . . ?

SOLUTION

Noticing that the difference between consecutive numbers in the sequence is 7, we may be tempted to say that the next term is $29 + 7 = 36$. Is this conclusion guaranteed? No! Another sequence in which numbers differ by 7 are dates of a given day of the week. For instance, the dates of the Fridays in the year 2016 are (January) 1, 8, 15, 22, 29, (February) 5, 12, 19, 26, ... Therefore, the next number in the sequence 1, 8, 15, 22, 29, . . . might be 5. Without further information, we cannot determine the next number in the given sequence. We can only use inductive reasoning and give one or more *possible* answers.

Topic X Sudoku: Logic in the Real World

Throughout history, people have always been attracted to puzzles, mazes, and brainteasers. Who can deny the inherent satisfaction of solving a seemingly unsolvable or perplexing riddle? A popular new addition to the world of puzzle solving is *sudoku,* a numbers puzzle. Loosely translated from Japanese, *sudoku* means "single number"; a sudoku puzzle simply involves placing the digits 1 through 9 in a grid containing 9 rows and 9 columns. In addition, the 9 by 9 grid of squares is subdivided into nine 3 by 3 grids, or "boxes," as shown in Figure 1.16.

The rules of sudoku are quite simple: Each row, each column, and each box must contain the digits 1 through 9; and no row, column, or box can contain 2 squares with the same number. Consequently, sudoku does not require any arithmetic or mathematical skill; sudoku requires logic only. In solving a puzzle, a common thought is "What happens if I put this number here?"

Like crossword puzzles, sudoku puzzles are printed daily in many newspapers across the country and around the world. Websites containing sudoku puzzles and strategies provide an endless source of new puzzles and help. See Exercise 74 to find links to popular sites.

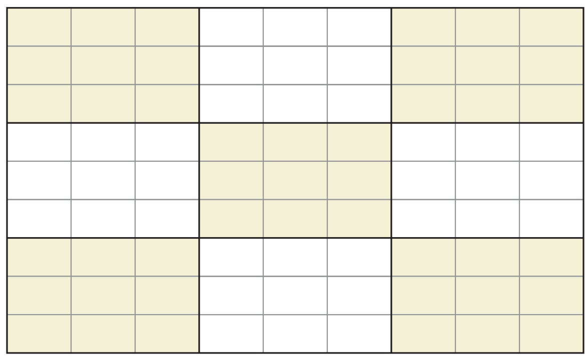


Figure 1.16 A blank sudoku grid.